Elasticity

Q:→ Derive an expression for bending moment of beam

Solⁿ:→ Beam is defined as a bar having uniform cross section area, this bar has length (l), width (b) and height (h). Now consider a beam with rectangular cross-sectional area, this beam is bent EF is known as neutral axis.

Neutral axis:-

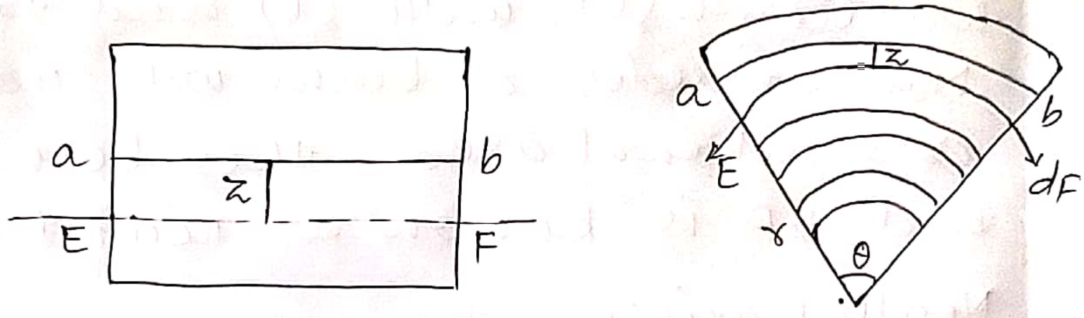
The line of intersection of neutral surface with the plane of bending is called neutral axis, there becomes no change in the length of the wire present on the neutral surface. This rectangular beam may be considered to be made up of a large no. of filaments present above the neutral surface suffer elongation and hence there acts a tensile force and those present below the neutral surface contracts, meaning thereby that, there acts a compressive force.

Plane of bending:-

The plane in which bending occurs is known as plane of bending.

Neutral surface :-

The surface which is perpendicular to plane of bending and bisects the beam is called neutral surface.



Initially, ab = EF when the beam is bent.

$$\theta = EF/r$$

$$\therefore EF = r\theta \quad \text{—— (1)}$$

$$\theta = \frac{a'b'}{(r+z)}$$

$$\therefore a'b' = r\theta + z\theta \text{ —— (2)}$$

Now, change in the length of the filament:

$$\cancel{r\theta} + z\theta - \cancel{r\theta} = z\theta$$

$\therefore$ Longitudinal strain in this filament

$$= z\theta / r\theta$$

$$= z/r$$

Longitudinal strain in this filament

$$= \frac{z\theta}{r\theta}$$

$$= \frac{z}{r}$$

Longitudinal stress $= \frac{dF}{dA}$

$$Y = \frac{dF}{dA} \times \frac{r}{z}$$

$$= \frac{r\,dF}{z\,dA}$$

$$\therefore dF = \frac{Yz\,dA}{r}$$

<u>Torque about the neutral axis :→</u>

$$d\tau = \frac{Yz^2 dA}{r}$$

Take a cross-section of this beam, the line lies on the neutral surface. Take two elements having area dA and symmetrically located about ef at a distance z from ef. Restoring torque about ef due to this couple

$$d\tau = \frac{2Yz^2 dA}{r}$$

Hence restoring couple about ef due to all couples,

$$\tau = \frac{2Y}{r}\int z^2 dA$$

$$\Rightarrow \quad \tau = \frac{Y A K^2}{r}$$

Where,

Y = Young's modulus of elasticity of the beam.

A = Cross-sectional area of beam.

r = Radius of curvature of neutral axis.

K = Geometrical radius of gyration about ef.

Note :- Geometrical moment of inertia is numerically equal to the moment of inertia, when $\sigma = M/A$ is equal to unity.