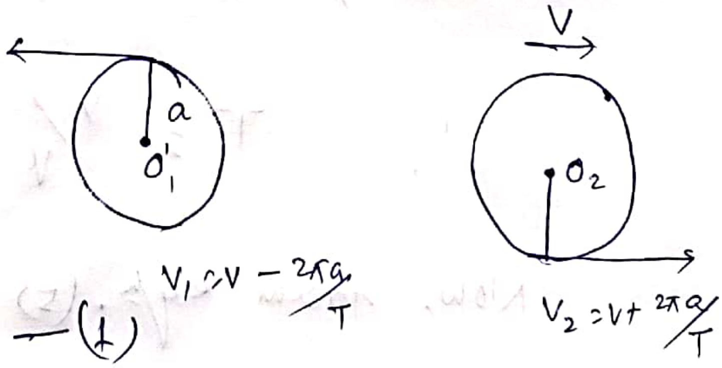
Q:- Describe ripple and gravity wave. Describe ripple method to determine surface tension of a liquid.

Soln.→ When a wave is produce on the surface of a liquid, the velocity of the wave is governed by the gravity force as well as surface tension. so it is called gravity and ripple wave. If the amplitude of the wave is small, then each particle on the surface of the liquid is assumed to describe it's motion on vertical circle. The time interval in which each particle makes one round trip on the circle, the wave advances at a distance λ. If the wave is propagating from left to right the particles describe anticlockwise circular path.

If V = velocity of the wave

T = time period then

velocity of the particle at rest



$$= V_1 = V - \frac{2\pi a}{T} \quad \text{---} (1)$$

Where, a = radius of circular path.

The velocity of particle at V_2 =

$$V + \frac{2\pi a}{T} \longrightarrow (2)$$

But

$$V_2^2 = V_1^2 + 2g(2a) = V_1^2 + 4ga$$

$$\Rightarrow V_2^2 - V_1^2 = 4ga \quad \text{--- (3)}$$

Putting the value of V_1 and V_2 from (1) and (2), we get,

$$V_2^2 - V_1^2 = \left(V + \frac{2\pi a}{T}\right)^2 - \left(\frac{V - 2\pi a}{T}\right)^2$$

$$= \frac{8\pi a V}{T} \quad \text{--- (4)}$$

So, from eqn. (3) & (4) we have,

$$\frac{8\pi a V}{T} = 4ga$$

$$\Rightarrow V = gT/2\pi \quad \text{--- (5)}$$

But, $V = \lambda/T$

$$\therefore T = \lambda/V \quad \text{--- (6)}$$

Now, from eqn. (5) & (6) we get,

$$V = g/2\pi \cdot \lambda/V$$

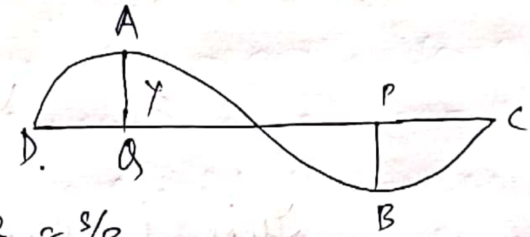
$$\Rightarrow V^2 = g\lambda/2\pi \quad \text{--- (7)}$$

Now, Suppose that the wave is travelling along x-axis, then

$$y = a\sin \frac{2\pi}{\lambda}(vt - x)$$

Due to gravity force the pressure at Q point will be $\rho g y$.

Due to surface tension the excess pressure at A point will be $= S/R$



where, S = Surface tension of liquid and

R = radius of curvature at A point.

Therefore pressure at Q point.

$$= \left(\rho g y - S/R\right)$$

But, $\frac{1}{R} = \frac{\partial^2 y}{\partial x^2}$

$$= -\left(\frac{2\pi}{\lambda}\right)^2 y$$

So, $$P = \rho g y + S \frac{4\pi^2}{\lambda^2} y$$

$$= \rho y \left(g + \frac{4\pi^2 S}{\rho \lambda^2}\right)$$

Thus, the effect of surface tension is to increase the accln. due to gravity

$$g_1 = g + \frac{4\pi^2 S}{\rho \lambda^2} \quad \text{—— (7)}$$

Therefore, when we consider the effect of surface tension, then

$$V = \sqrt{g_1\lambda/2\pi}$$

$$\Rightarrow v = \sqrt{\lambda/2\pi\left(g + \frac{4\pi^2 S}{\rho\lambda^2}\right)}$$

$$\Rightarrow v = \sqrt{g\lambda/2\pi + \frac{2\pi S}{\rho\lambda}} \quad \text{--- (8)}$$

This is the expression for velocity of gravity and ripple wave for minimum value λ, we have

$\left[\frac{dv}{d\lambda}\right] = 0$ this gives

$$\lambda m = \sqrt{\frac{4\pi^2 S}{\rho g}} \quad \text{--- (9)}$$

Thus, the minimum velocity of ripple wave is given as

$$V_{min.} = \left(\frac{4Sg}{\rho}\right)^{1/4}$$

Now, if the value of λ is smaller than λ_m then,

$$V = \sqrt{2\pi S/\rho\lambda}$$

It is called the ripple wave. If $\lambda > \lambda_m$ then,

$$V = \sqrt{g\lambda/2\pi}$$

This is called the gravity wave.