Reduced equation of state and the law of corresponding state:→

If the Pressure, volume and temperature of a gas are expressed in terms of the critical pressure, volume and temperature, then we may write:

$$P = \pi P_c, \quad V = \phi V_c, \quad T = \theta T_c \quad \text{—①}$$

Thus the quantities π, ϕ and θ may be defined as.

$$\pi = \frac{P}{P_c}, \quad \phi = \frac{V}{V_c} \text{ and } \theta = \frac{T}{T_c}.$$

The quantity π, ϕ and θ are called reduced Pressure, reduced volume and reduced temperature. respectively.

Substituting the values of P, V and T given by equⁿ ①, in Vander Waal's equation.

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

we get $$\left(\pi P_c + \frac{a}{\phi^2 V_c^2}\right)(\phi V_c - b) = R\theta T_c$$

Further substituting the values of P_c, T_c and V_c in terms of Vander Waal's constant viz.

$$V_c = 3b$$

$$P_c = \frac{a}{27b^2} \text{ and } T_c = \frac{8a}{27Rb}$$

we get $$\left(\pi \frac{a}{27b^2} + \frac{a}{\phi^2 9b^2}\right)(\phi 3b - b) = R\theta \frac{8a}{27Rb}.$$

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or $\left(\pi a + \frac{3a}{\phi^2}\right)(3b\phi - b) = 8\theta ab$

or $\boxed{\left(\pi + \frac{3}{\phi^2}\right)(3\phi - 1) = 8\theta}$

This equation which relates the reduced Pressure, Volume and temperature is called the reduced equation of state. The most important point about this equation is that this doesnot involve the constant a, b and R and hence is applicable to all substances in the liquid and the gaseous state. Another important conclusion drawn from the above equation is as follows.

If two substances have the same reduced pressure and the same reduced temperature, they must have the same reduced volume.

The above statement is called the law of corresponding state and the substances having the same reduced Pressure and same reduced temperature and hence the same reduced volume are said to be in the corresponding states.

Relation between critical constant (P_c, V_c and T_c)

We know that

$V_c = 3b$ — (i)

$P_c = \frac{a}{27b^2}$ — (ii)

$T_c = \frac{8a}{27Rb}$ — (iii)

To obtain a relation ship between P_c, V_c and T_c we have to eliminate a and b from equations (i), (ii) and (iii). This may be done as follows.

$b = \frac{V_c}{3}$ (Put the value of b in equn (ii)) (iv)

$P_c = \frac{a}{27\left(\frac{V_c}{3}\right)^2} = \frac{a}{3V_c^2}$

$a = 3P_cV_c^2$ — (v)

Substituting the values of (a) and (b) from equn (iv) and (v) in equn (iii)

$T_c = \frac{8 \times 3P_cV_c^2}{27R \times V_c/3}$

$T_c = \frac{8}{3}\frac{P_cV_c}{R}$

$\therefore \boxed{P_cV_c = \frac{3}{8}RT_c}$