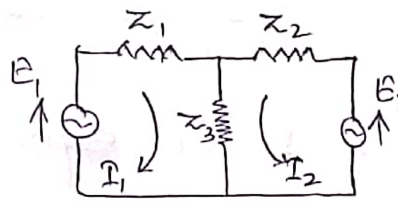
Superposition theorem:

In any linear network containing impedances and energy sources, the current flowing in any element is the vector sum of the currents that are separately caused to flow in that element by each energy source.



fig(1) (a) Two mesh network

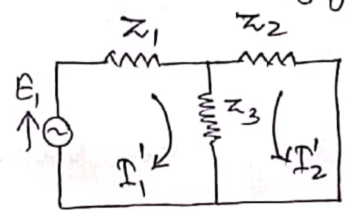
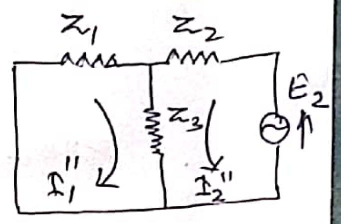


fig (1)(b) Two mesh network when E_2 is removed.



fig(1)(c) Two mesh network when E_1 is removed.

Consider the two voltage sources E_1 and E_2 are three impedances Z_1, Z_2 and Z_3 as shown in figure. Let I_1 and I_2 be the two mesh currents than the mesh equation can be written as,

$$E_1 = (Z_1 + Z_3) I_1 + Z_3 I_2 \quad \text{—— (1)}$$

$$E_2 = (Z_3 I_1) + (Z_2 + Z_3) \cdot I_2 \quad \text{—— (2)}$$

From equation (2) we get,

$$E_2 - (Z_2 + Z_3) \cdot I_2 = Z_3 I_1$$

$$\Rightarrow I_1 = \frac{E_2}{Z_3} - \frac{(Z_2 + Z_3) \cdot I_2}{Z_3}$$

Putting the value of I_1 in equation (1) we get,

$$E_1 = (Z_1 + Z_3) \left[\frac{E_2}{E_3} - \frac{(Z_2 + Z_3)}{Z_3} I_2 \right] + Z_3 I_2$$

$$\Rightarrow E_1 = \frac{(Z_1 + Z_3) E_2}{Z_3} - \frac{(Z_1 + Z_3)(Z_2 + Z_3) I_2}{Z_3} + Z_3 I_2$$

$$E_1 = \frac{(Z_1+Z_3)E_2}{Z_3} - \frac{(Z_1Z_2+Z_2Z_3+Z_1Z_3)\cdot I_2}{Z_3}$$

or $$I_2 = -\frac{E_1 Z_3}{Z_1Z_2+Z_2Z_3+Z_1Z_3} + \frac{(Z_1+Z_3)E_2}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{——— (3)}$$

Substituting the value of I_2 in eqn. (1) we get,

$$I_1 = \frac{(Z_2+Z_3)E_1}{Z_1Z_2+Z_2Z_3+Z_1Z_3} - \frac{Z_3E_2}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{—— (4)}$$

Now, considering the circuit of figure (1)(b) we have the mesh equations,

$$E_1 = (Z_1+Z_3)I_1' + Z_3I_2' \quad \text{——— (5)}$$

$$0 = Z_3I_1' + (Z_2+Z_3)I_2' \quad \text{——— (6)}$$

Solving eqn. (5) and (6) we get,

$$I_2' = \frac{-Z_3E_1}{[Z_1Z_2+Z_2Z_3+Z_1Z_3]} \quad \text{—— (7)}$$

$$I_1' = \frac{(Z_2+Z_3)E_1}{(Z_1Z_2+Z_2Z_3+Z_1Z_3)} \quad \text{—— (8)}$$

Referring to circuit of figure (1)(c) the mesh equations are.

$$0 = (Z_1+Z_3)I_1'' + Z_3I_2'' \quad \text{—— (9)}$$

$$E_2 = Z_3I_1'' + (Z_2+Z_3)I_2'' \quad \text{—— (10)}$$

Solving these equations, we have,

$$I_2'' = \frac{(Z_1+Z_3)E_2}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{--- (11)}$$

$$I_1'' = \frac{-Z_3E_2}{Z_1Z_2+Z_2Z_3+Z_1Z_3} \quad \text{--- (12)}$$

From equation (3), (4), (7), (8) (11) and (12) we get,

$$I_1 = I_1' + I_1'' \text{ and}$$

$$I_2 = I_2' + I_2''$$

This proves the theorem.